Integrated and collaborative routing problems

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Malta, May 30th, 2019
The framework

The Internet of Things

Big Data

Sharing Economy

Integrated and collaborative routing
Directions in routing problems

- Integrated
- Collaborative
- Data-driven
Integrated direction

Vehicle routing

Location

Network design

Production scheduling

Inventory management
Inventory routing problems

Routing problems over time
Pick-up and delivery inventory routing problem

- Pick-up
- Delivery

Given availability and demand over time

Archetti, Christiansen, Speranza, EJOR, 2018
Pick-up and delivery inventory routing problem
Pick-up and delivery inventory routing problem

Day t+1
Pick-up and delivery inventory routing problem

- Pick-up customers – daily quantity made available
- Delivery customers – daily demand
- One vehicle with capacity Q
- Maximum and minimum inventory level at customers
- The depot is a warehouse where goods can be stored

\[
\text{Min routing cost } + \text{ inventory holding cost}
\]
Pick-up and delivery inventory routing problem

Variables:

- Quantity (horizon x customers) – continuous
- Inventory level (horizon x customers) - continuous
- Visit schedule (horizon x customers) - binary
- Edge traversal (horizon x customer\(^2\)) – binary
- Load (horizon x customer\(^2\)) – continuous

Objective function:

Min routing + inventory holding costs

Constraints:

- Inventory constraints
- Vehicle capacity constraints
- Routing constraints
- Load constraints
Pick-up and delivery inventory routing problem

640 instances with varying:

- vehicle capacity: $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$
- horizon: 3 or 6
- inventory cost: high or low
- number of customers: up to 50

473 instances solved to optimality
1.22 average optimality gap

Improved branch-and-cut algorithm

Archetti, Boccia, Sforza, Speranza, Sterle, submitted

538 instances solved to optimality
0.89 average optimality gap
133 improved solutions
Pick-up and delivery inventory routing problem

- Integrated policy
- Sequential policy: each delivery customer applies \((s,S)\) vehicle routing problems

<table>
<thead>
<tr>
<th></th>
<th>% total cost (average)</th>
<th>% total cost (max)</th>
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Directions in routing problems

- Integrated
- Collaborative
- Data-driven
Routing among loading/unloading areas

Mor, Speranza, Viegas, in preparation, 2019
Routing among loading/unloading areas
Routing among loading/unloading areas
Routing among loading/unloading areas

Each vehicle makes a reservation of the L/U areas

- Windows of availability for the following vehicles

- Variant of TSP with multiple time windows
TSP with multiple time windows

\[
\begin{align*}
\min & \ T_{|U|+1} - T_0 \\
\sum_{u \in U} & \ x_{0u} = 1 \\
\sum_{u \in U} & \ x_{u(|U|+1)} = 1 \\
\sum_{i \in \{0\} \cup U} & \ x_{iu} = \sum_{i \in U \cup \{|U|+1\}} x_{ui} = 1, \quad u \in U, \\
(T_i + s_i + t_{ij} - T_j) & \leq M(1 - x_{ij}), \quad i \in \{0\} \cup U, j \in U \cup \{|U|+1\}, \\
T_u & \geq W_{u,h}^a y_{u,h}, \quad u \in U, h \in H_u, \\
T_u + s_u & \leq W_{u,h}^b + M(1 - y_{u,h}), \quad u \in U, h \in H_u, \\
\sum_{h \in H_u} & \ y_{u,h} = 1, \quad u \in U, \\
x_{ij} & \in \{0, 1\}, \quad i \in \{0\} \cup U, j \in U \cup \{|U|+1\}, \\
T_i & \geq 0, \quad i \in \{v_k, v_k+1\} \cup U, \\
y_{u,h} & \in \{0, 1\}, \quad u \in U, h \in H_u.
\end{align*}
\]
Routing among loading/unloading areas

Fixed starting time of each route

\[ t_{0i} = 0 \quad \text{i} \in U \cup \{ |U| + 1 \} \]
\[ t_{i(|U|+1)} = 0 \quad \text{i} \in \emptyset \cup U \]
\[ T_0 = 0 \]
\[ s_0 = h \]

Variable starting time of each route

\[ s_0 = 0 \]
Routing among loading/unloading areas
Routing among loading/unloading areas

![Graph showing time (minutes) vs vehicles for different routing methods.

- Orange line: Indep TSP
- Blue line: SEQ fixed ST
- Red line: SEQ variable ST

The graph illustrates the time taken in minutes for different numbers of vehicles, with the time increasing as the number of vehicles increases. Indep TSP shows a more linear increase, while SEQ fixed ST and SEQ variable ST maintain a relatively constant time with a slight increase as the number of vehicles grows.]
Directions in supply chain management

Integrated

Collaborative

Data-driven
VRP with release dates

crossdocking

Arrival time of the goods for customer $i$ = release date of customer $i$
VRP with release dates

8:30am

Release date = 8:30am
VRP with release dates

Release date = 8:40 am
Release date = 8:30 am
Release date = 8:40 am
TSP with release dates

Min
Maximum completion time
= Traveling time + Waiting time at the depot

Archetti, Feillet, Mor, Speranza, EJOR, 2018
TSP with release dates

**Property:** There exists an optimal solution without waiting time after the start of the first route

\[
t(S^*) \geq \frac{r_n + d_{TSP}}{2}
\]

A lower bound:  

An approximation algorithm:  

- Apply Christofides’ algorithm  
- Start the TSP tour obtained at time \( r_n \)

Performance guarantee: 2.5
TSP with release dates

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Iterated local search
Iterated local search with a MILP operator
**TSP with release dates**

Myopic: visit customers when they become available

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50 customers: gaps with respect to best known solution
Directions in routing problems

Integrated and collaborative routing

Integrated

Data-driven

Collaborative
TSP with stochastic release dates

Arrival time at the warehouse often not known

Information may become available over time
TSP with stochastic release dates

The release date of a customer (arrival time of a truck) may be:

- known (reliable)
- static (random but the distribution does not change over time)
- dynamic (random and the distribution changes over time)
TSP with stochastic release dates

Static and dynamic release dates
TSP with stochastic release dates

$t = 0$

Any unvisited customer?

yes

Optimization model

is $t = \tau_{\text{start}}^{\text{next order}}$

yes

Execute route

Update $t$ as ending time of executed route

Remove visited customers

no

Solution value is $t$

no

Update $t$ as next decision epoch
TSP with stochastic release dates

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Time to perform 1 iteration of the Iterated Local Search
Directions in routing problems

Integrated

Collaborative

Data-driven
The shared customer collaboration VRP

- Each company has its depot and fleet
- There is a subset of **shared** customers
- Companies are willing to **share** the service of some customers with other companies in order to decrease their cost

**Horizontal**
**Between competitors**
**Order sharing**

_Fernàndez, Roca-Riu, Speranza, EJOR, 2018_
The shared customer collaboration VRP
The shared customer collaboration VRP
Analysis

\[ z^* (\text{SCC-VRP}) \geq \max_{r \in C} z^* (\text{VRP}_r) \]

\[ z^* (\text{SCC-VRP}) \geq \frac{z^*(m-\text{VRP})}{m} \]

All depots are co-located
All carriers have 1 co-located customer

If it is guaranteed that in the collaboration the profit of each company does not decrease with respect to any sub-coalition, the solution belongs to the core of the game.
Formulations

Vehicle formulation

\[ x_{ij}^k \] arc \( i,j \) for vehicle \( k \)

\[ z_{irs}^k \] customer \( i \) from carrier \( r \) to \( s \), with vehicle \( k \)

Load formulation

\[ x_{ij}^r \] arc \( i,j \) by carrier \( r \)

\[ z_{irs} \] customer \( i \) from carrier \( r \) to \( s \)

\[ l_{ij}^{rh} \] load on arc \( i,j \) by carrier \( r \) for customer \( h \)
Solution approach

Vehicle formulation

Branch & Cut
+ Cover inequalities
+ Capacity-cut inequalities
+ Symmetry breaking constraints

Load formulation

Branch & Cut
+ Connectivity constraints
+ Capacity-cut inequalities
+ Symmetry breaking constraints
Test instances

**S1** — adapted from Cordeau, Gendreau, Laporte, 1997
12 instances for the MDVRP
18-30 customers

**S2** — randomly generated
100 random/clustered instances
10-30 customers
## Comparison between VF and LF (S1)

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2 hours
Savings

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<th>$S_{2R}$ Random</th>
<th>$S_{2C}$ Clustered</th>
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Individual (without collaboration) solutions always optimal
Directions in routing problems

Integrated

Collaborative

Data-driven
Crowd driving: Occasional drivers

Archetti, Savelsbergh, Speranza, EJOR, 2016
Occasional drivers
Occasional drivers

Costs:
- Routing cost for regular drivers
- Compensation to occasional drivers

Objective:
Minimize the sum of the cost of regular drivers (routing cost) and occasional drivers (compensation)
Occasional drivers

- Behaviour of occasional drivers
- Compensation schemes
- Objective: Minimize the sum of the cost incurred by regular drivers (routing cost) and occasional drivers (compensation)
 Behaviour of occasional drivers

Integrated and collaborative routing
Compensation schemes

1. Proportional to the distance of the on-line customer

2. Proportional to the detour
VRP with occasional drivers

\[
\begin{align*}
\text{min} & \quad \sum_{(i,j) \in A} c_{ij} x_{ij} + \sum_{i \in C} \sum_{k \in K} p_{ik} w_{ik} \\
\text{subject to} & \quad \sum_{j | (i,j) \in A} x_{ij} = \sum_{j | (j,i) \in A} x_{ji} = z_i \quad i \in C \\
& \quad \sum_{j | (0,j) \in A} x_{0j} - \sum_{j | (j,0) \in A} x_{j0} = 0 \\
& \quad \sum_{j | (j,i) \in A} y_{ji} - \sum_{j | (i,j) \in A} y_{ij} = \begin{cases} d_i z_i & i \in C \\ \sum_{i \in C} -d_i z_i & i = 0 \end{cases} \\
& \quad y_{ij} \leq Q x_{ij} \quad (i, j) \in A \\
& \quad y_{i0} = 0 \quad i \in C \\
& \quad w_{ik} \leq \beta_{ik} \quad i \in C, k \in K \\
& \quad \sum_{i \in C} w_{ik} \leq 1 \quad k \in K \\
& \quad \sum_{k \in K} w_{ik} + z_i = 1 \quad i \in C
\end{align*}
\]

depends on compensation scheme

- Exact
- Matheuristic
VRP with occasional drivers

Without occasional drivers

Integrated and collaborative routing
VRP with occasional drivers

Compensation scheme proportional to detour
VRP with occasional drivers

Compensation scheme proportional to detour

(lower compensation)
## Savings

<table>
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<tr>
<th></th>
<th>% cost reduction w.r.t. VRP</th>
<th>% routes reduction w.r.t. VRP</th>
<th>%OD used</th>
<th>% OD cost w.r.t. total cost</th>
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Conclusions

Our models and methods

• evolve with the technology

• contribute to the technology